A family of synchrosqueezing transforms for multicomponent signal analysis

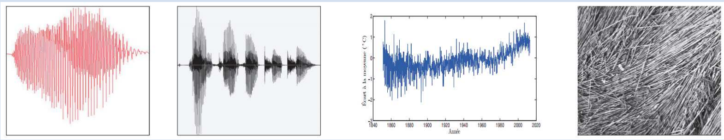
Duong-Hung Pham

A FAMILY OF SYNCHROSQUEEZING TRANSFORMS FOR MULTICOMPONENT SIGNALS ANALYSIS

Duong-Hung PHAM - Joint work with Sylvain MEIGNEN
Jean Kuntzmann Laboratory, University of Grenoble-Alpes and CNRS, France.
(Email:duong-hung.pham@imag.fr and sylvain.meignen@imag.fr)

Introduction

- Time-frequency analysis of multicomponent signals (MCS).
- MCS in real life:



- The SynchroSqueezed Transform (SST) [1] has two purposes:
  - sharpen the time-frequency (TF) representation given by Short-Time Fourier Transform (STFT)
  - reconstruct automatically the modes making up the signal.
- The goal of this research: put forward a generalization of SST using a new local estimate of instantaneous frequency (IF) $\Longrightarrow$ achieve a highly concentrated TF representation for a larger class of MCSs + reconstruct their modes with a high accuracy.

Multicomponent signal (MCS)

- Superposition of AM - FM modes: $f(t) = \sum_{k=1}^{K} f_k(t)$ *with* $f_k(t) = A_k(t)e^{i2\pi\phi_k(t)}$, for $K \in \mathbb{N}$, $A_k(t) > 0, \phi'_k(t) > 0$ and $\phi'_{k+1}(t) > \phi'_k(t)$ for $\forall t$.
- Hypothesis: all f_ks are well separated in frequency, i.e. $|\phi'_{k+1}(t) - \phi'_k(t)| \geq 2\Delta$ for $\forall t$.

STFT

- Fourier transform (FT) of a signal $f \in L^1(\mathbb{R})$: $\hat{f}(\eta) = \int_{\mathbb{R}} f(t)e^{-i2\pi\eta t}dt$.
- A signal $f \in L^1(\mathbb{R})$ and a window $g \in \mathcal{S}(\mathbb{R})$:

$$V_f^g(t,\eta) = \int_{\mathbb{R}} f(\tau)g^*(\tau - t)e^{-2i\pi\eta(\tau-t)}d\tau.$$

Reassignment methods

- Reassignment operators:
  - Local group delay (GD):

$$\hat{\tau}_f(t,\eta) = t - \frac{1}{2\pi}\partial_\eta\left\{\arg(V_f^g(t,\eta))\right\}$$

.

  - Local instantaneous frequency:

$$\hat{\eta}_f(t,\eta) = \frac{1}{2\pi}\partial_t\left\{\arg(V_f^g(t,\eta))\right\}$$

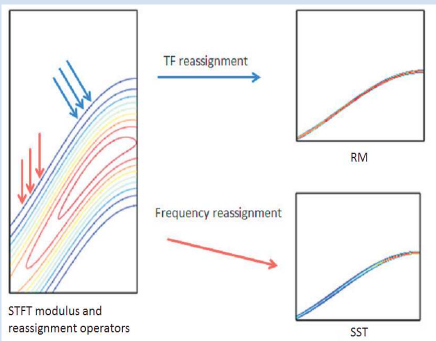
- Standard reassignment (RM):
  - Oblique mapping: $(t,\eta) \mapsto (\hat{\tau}_f, \hat{\eta}_f)$.
  - Operator:

$$R_f^g(t,\omega) = \int\int_{\mathbb{R}^2} |V_f^g(\tau,\eta)|^2 \times \delta(\omega - \hat{\omega}_f(\tau,\eta))\,\delta(t - \hat{\tau}_f(\tau,\eta))\,d\eta d\tau.$$

  - Ideal TF representation of linear chirps.
  - Non reconstruction.
- SST:
  - Vertical mapping: $(t,\eta) \mapsto (t, \hat{\eta}_f)$.
  - Operator:

$$T_f^g(t,\omega) = \frac{1}{g^*(0)}\int_0^\infty V_f^g(t,\eta)\delta(\omega - \hat{\omega}_f(t,\eta))\,d\eta$$

  - Ideal TF representation of pure waves.
  - Reconstruction.



References

[1] G. Thakur and H.-T. Wu, "Synchrosqueezing-based recovery of instantaneous frequency from nonuniform samples." *SIAM J. Math. Analysis*, vol. 43, no. 5, pp. 2078–2095, 2011.

[2] D.-H. Pham and S. Meignen, "High-order synchrosqueezing transform for multicomponent signals analysis - with an application to gravitational-wave signal," *IEEE Transactions on Signal Processing*, vol. 65, no. 12, pp. 3168–3178, June 2017.

Acknowledgements

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Toward to high-order SST (SSTN)

- Let $f \in L^2(\mathbb{R})$, frequency modulation operators $\tilde{q}_{\eta,f}^{[p,N]}$ of $\phi^{(p)}(t)/(p-1)!$ for $p = 2,3,4$ and $N = 4$ are:

$$\tilde{q}_{\eta,f}^{[4,4]} = G_4\left(V_f^{t^{0\ldots6}g}, V_f^{t^{0\ldots3}g'}\right),$$

$$\tilde{q}_{\eta,f}^{[3,4]} = G_3\left(V_f^{t^{0\ldots4}g}, V_f^{t^{0\ldots2}g'}\right) - \tilde{q}_{\eta,f}^{[4,4]}G_{3,4}\left(V_f^{t^{0\ldots5}g}\right),$$

$$\tilde{q}_{\eta,f}^{[2,4]} = G_2\left(V_f^{t^{0\ldots2}g}, V_f^{t^{0\ldots1}g'}\right) - \tilde{q}_{\eta,f}^{[3,4]}G_{2,3}\left(V_f^{t^{0\ldots3}g}\right) - \tilde{q}_{\eta,f}^{[4,4]}G_{2,4}\left(V_f^{t^{0\ldots4}g}\right),$$

where $G_p\left(V_f^{t^{0\ldots m}g}, V_f^{t^{0\ldots n}g'}\right)$ is a function of $V_f^{t^l g}$ for $l = 0,\ldots,m$ and $V_f^{t^l g'}$ for $l = 0,\ldots,n$ while $G_{p,j}\left(V_f^{t^{0\ldots m}g}\right)$ is associated with coefficient $\tilde{q}_{\eta,f}^{[j,N]}$ in the computation of $\tilde{q}_{\eta,f}^{[p,N]}$ for $p \neq j$.

- IF estimate of order 4 is:

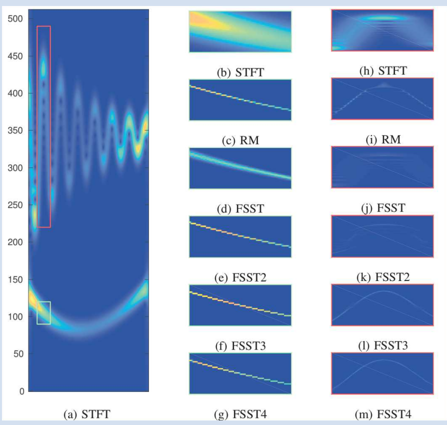
$$\tilde{\omega}_{\eta,f}^{[4]}(t,\eta) = \tilde{\omega}_f(t,\eta) + \tilde{q}_{\eta,f}^{[2,4]}(t,\eta)\left(-x_{2,1}(t,\eta)\right) + \tilde{q}_{\eta,f}^{[3,4]}(t,\eta)\left(-x_{3,1}(t,\eta)\right) + \tilde{q}_{\eta,f}^{[4,4]}(t,\eta)\left(-x_{4,1}(t,\eta)\right).$$

  - Exact IF estimate for a polynomial chirp of order 4.
  - $\tilde{\omega}_{\eta,f}^{[2]}$ is obtained by neglecting $\tilde{q}_{\eta,f}^{[3,4]}$ and $\tilde{q}_{\eta,f}^{[4,4]}$.

- Synchrosqueezing operator of order N (SSTN) is:

$$T_{N,f}^g(t,\omega) = \frac{1}{g^*(0)}\int_0^\infty V_f^g(t,\eta)\delta\left(\omega - \hat{\omega}_{\eta,f}^{[N]}(t,\eta)\right)d\eta.$$

Numerical results



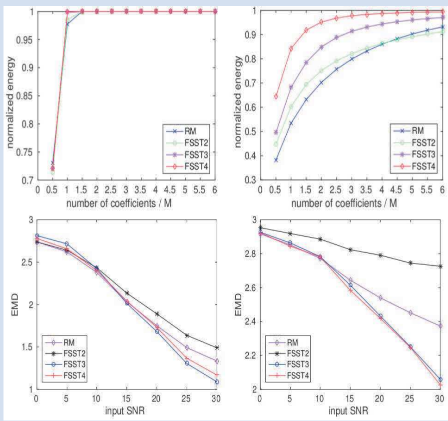
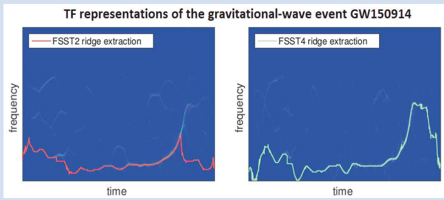


TABLE I
PERFORMANCE OF MODE RECONSTRUCTION (MEASURED BY SNR)

	FSST2	FSST3	FSST4
Mode f_1	17.8	25.7	**28.8**
Mode f_2	1.73	3.62	**6.87**
MCS f	3.57	5.57	**8.82**



Conclusion

- SSTN: a powerful tool for analysis of MCS containing very strongly modulated AM-FM modes.
- Combination of a sharp representation (like reassignment) and a reconstruction (like classical ridge analysis)
- An interesting application on gravitational-wave signal.

Current and future works

- Theoretical analysis of SSTN when applied to noisy signals and when the type of noise is non Gaussian.
- Extension to 2 or 3 dimensions (with monogenic SST).
- More applications for real-life signals (detection, monitoring, etc.).